SI 2013 Mark Scheme Q1

	Working	Mark	Guidance
(i)	$y=\sqrt{x} \;\Rightarrow\; y^2+3y-\frac{1}{2}=0$	**B1**	for a correct quadratic eqn. in y or $\sqrt{x}$
	$(2y+3)^2=11$	**M1**	for use of a method for solving a quadratic eqn. (compl[g]. the square, formula, etc.) If candidate fails to obtain a numerical answer for y (correct or not) then **M0**
	$y=\dfrac{-3\pm\sqrt{11}}{2}$	**A1**	
	$y\ge 0 \;\Rightarrow\; \sqrt{x}=\dfrac{\sqrt{11}-3}{2}$	**M1**	for clearly choosing the correct root: **FT** *provided* they have 1 $+_{ve}$ and 1 $-_{ve}$ root to choose from
	$x=\left(\dfrac{\sqrt{11}-3}{2}\right)^2$ or $\dfrac{20-6\sqrt{11}}{4}$ or $5-\frac{3}{2}\sqrt{11}$	**A1**	**5**
(ii) (a)	$y=\sqrt{x+2}$	**M1**	for clear indication of this substitution (or equivalent)
	$y^2+10y-24=0$	**A1**	for a correct quadratic
	y or $\sqrt{x+2}=-12,\ 2$	**M1**	for solution method of a suitable quadratic
	$y\ge 0 \;\Rightarrow\; \sqrt{x+2}=2$	**M1**	for choosing valid root: **FT** *provided* they have 1 $+_{ve}$ and 1 $-_{ve}$ root to choose from
	$x=2$	**A1**	**5**
(ii) (b)	$y=\sqrt{2x^2-8x-3}$	**M1**	for clear indication of this substitution (or equivalent)
	$y^2+2y-15=0$	**A1**	for a correct quadratic
	y or $\sqrt{2x^2-8x-3}=-5,\ 3$	**M1**	for solution of a suitable quadratic
	$y\ge 0 \;\Rightarrow\; \sqrt{2x^2-8x-3}=3$	**M1**	for choosing valid root: **FT** *provided* they have 1 $+_{ve}$ and 1 $-_{ve}$ root to choose from
	$2x^2-8x-3=9 \;\Rightarrow\; x^2-4x-6=0$	**M1A1**	for obtaining and solving a quadratic eqn. in x; A1 for the correct quadratic
	$x=2\pm\sqrt{10}$	**A1**	**7**

$x = 2 \pm \sqrt{10}$ $\Rightarrow$ $x^2 = 14 \pm 4\sqrt{10}$ **M1** for checking attempt (for at least one of the answers found)

so $x^2 - 4x - 9 = -3$ & $2x^2 - 8x - 3 = 9$

$\Rightarrow$ (both cases) $-3 + \sqrt{9} = 0$ **A1A1** one for each clearly shown (with working)

ALTERNATIVELY For validity, $2x^2 - 8x - 3 \ge 0$ also **M1** i.e. $(x-2)^2 \ge \frac{11}{2}$ **A1**

Since $(x-2)^2 = 10 > \frac{11}{2}$ both solns. valid **E1**

3

ALTERNATIVES

(i) $3\sqrt{x} = \frac{1}{2} - x$ and squaring **M1** $x^2 - 10x + \frac{1}{4} = 0$ **A1** correct quadratic **M1** for solution of a suitable quadratic **A1** $x = 5 \pm \frac{3}{2}\sqrt{11}$

However, *both* these roots are positive, so the final mark will be **E1** for checking both, with working, and correctly discarding the unsuitable answer

(ii) (a) $10\sqrt{x+2} = 22 - x$ and squaring **M1** $x^2 - 144x + 284 = 0$ **A1** correct quadratic **M1** for solution of a suitable quadratic **A1** x = 142, 2

E1 for checking both, with working, and correctly discarding the unsuitable answer (e.g. x = 142 gives LHS > 0 but RHS < 0 would suffice)

(ii) (b) $\sqrt{2x^2 - 8x - 3} = 9 + 4x - x^2$ and squaring **M1** $x^4 - 8x^3 - 4x^2 + 80x + 84 = 0$ **A1** correct quartic

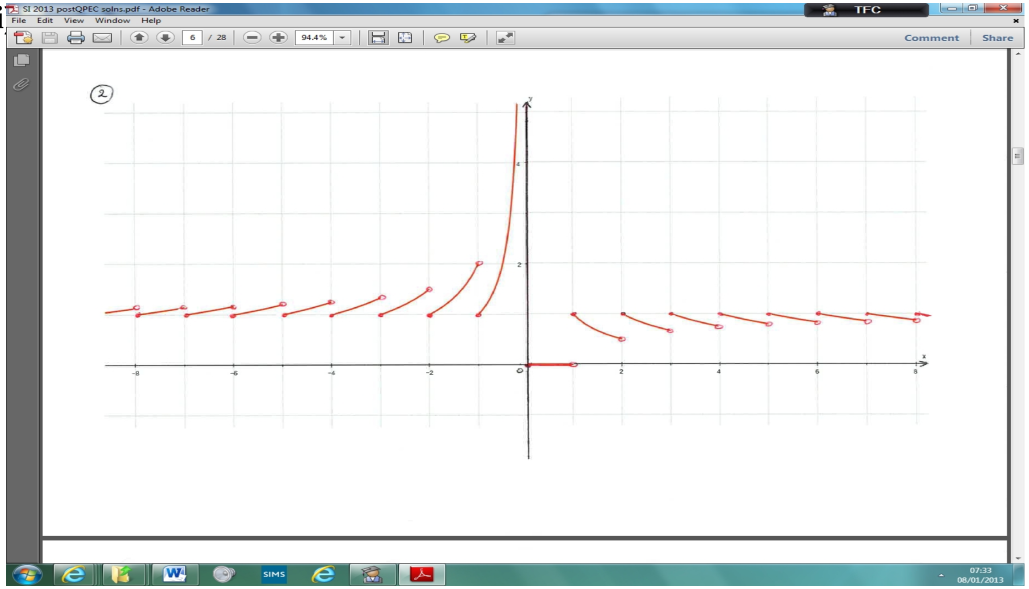
$(x-2)^4 - 28(x-2)^2 + 180 = 0$ **M1A1** $\Rightarrow$ $(x-2)^2 = 10, 18$ **M1A1**

Now $\sqrt{2}\sqrt{(x-2)^2 - \frac{11}{2}} = 13 - (x-2)^2$ $\Rightarrow$ $\frac{11}{2} \le (x-2)^2 \le 13$ **M1A1A1** so the only valid solutions arise from $(x-2)^2 = 10$ and $x = 2 \pm \sqrt{10}$ **A1**

However, I cannot see candidates making this approach work. **M1A1** for getting the correct quartic may be all they can reasonably get.
Attempts to find linear factors (by the *Factor Theorem*, for instance) will go nowhere.
Some may attempt to find a pair of quadratic factors: $(x^2 + Ax + B)(x^2 + Cx + D) \equiv x^4 + (A+C)x^3 + (AC + B + D)x^2 + (AD + BC)x + BD = 0$ and compare terms ($A + C = -8$, $AC + B + D = -4$, $AD + BC = 80$ and $BD = 84$) but I do not want them to have any marks unless they can get to (by guessing/verifying … divine inspiration?) $(x^2 - 4x - 6)(x^2 - 4x - 14)$, at which point I would award them the next **M1A1** & **M1A1**.
They now have four answers to check for and I would propose a **B1** for each correctly checked (with working) and accepted/rejected appropriately.

SI 2013 Mark Scheme Q2

(i



G1 Lots of "unit" segments

G1 LH / RH end clearly included / excluded

G1 Each segment looks like a portion of a reciprocal curve

G1 Essentially correct $0 \le x \le 3$

G1 Essentially correct $-3 \le x < 0$

Ignore endpoint issues for these last two Gs

5

(ii) Note that, for $n \le x < n+1$, $\lfloor x \rfloor = n$ so f(x) = $\frac{n}{x}$. Also, $\frac{n}{n+1} < \mathrm{f}(x) \le 1$ for $x > 0$, and f(x) ≥ 1 for $x < 0$, so …

f(x) = $\frac{7}{12}$ only in [1, 2). **E1** Sketch may show it so

$\mathrm{f}(x) = \frac{1}{x} = \frac{7}{12} \Rightarrow x = \frac{12}{7}$ **B1** B0 if extra answers appear

2

Similarly, $\frac{n}{n+1} > \frac{17}{24} \Rightarrow 24n > 17n + 17 \Rightarrow n > 2\frac{3}{7}$, i.e. $n \ge 3$; so f(x) = $\frac{17}{24}$ only in [1, 2) and [2, 3).

In [1, 2), $f(x) = \frac{1}{x} = \frac{17}{24} \Rightarrow x = \frac{24}{17}$ **B1**

In [2, 3), $f(x) = \frac{2}{x} = \frac{17}{24} \Rightarrow x = \frac{48}{17}$ **B1** Give max. B1 if extra answers appear

2

Now, for $x < 0$, $1 \le f(x) < \frac{n}{n+1}$, and $\frac{-n}{-n-1} < \frac{4}{3} \Rightarrow -4n - 4 > -3n \Rightarrow n < -4$; so $f(x) = \frac{4}{3}$ only in [–4, –3), [–3, –2), [–2, –1) and [–1, 0).

f(–3) = 1 so no solution in [–4, –3) **B1** Possibly implicitly, if just not there

In [–3, –2), $f(x) = \frac{-3}{x} = \frac{4}{3} \Rightarrow x = -\frac{9}{4}$ **B1**

In [–2, –1), $f(x) = \frac{-2}{x} = \frac{4}{3} \Rightarrow x = -\frac{3}{2}$ **B1**

In [–1, 0), $f(x) = \frac{-1}{x} = \frac{4}{3} \Rightarrow x = -\frac{3}{4}$ **B1** Give max. B1B1 if extra answers appear

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(iii) $\frac{n}{n+1} > \frac{9}{10}$ for $n > 9$ so …

$f(x_{max}) = \frac{9}{10}$ in [8, 9) **E1**

and $f(x) = \frac{8}{x} = \frac{9}{10} \Rightarrow x = \frac{80}{9}$ **B1** NB f(10) = 1, so $x \ne 10$

2

$f(x) = c$ has exactly n roots for …

for $x > 0$: $\frac{n}{n+1} < c \le \frac{n+1}{n+2}$ **B1B1** LHS; RHS

for $x < 0$: $\frac{n+1}{n} \le c < \frac{n}{n-1}$, $n \ge 2$ **B1B1** LHS; RHS

$c \ge 2$, $n = 1$ **B1**

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SI 2013 Mark Scheme Q3

(i)	$X * Y = Y * X \iff \lambda\mathbf{x} + (1-\lambda)\mathbf{y} = \lambda\mathbf{y} + (1-\lambda)\mathbf{x}$	**M1**	Including correct $Y * X = \lambda\mathbf{y} + (1-\lambda)\mathbf{x}$	
	$\iff (2\lambda - 1)(\mathbf{x} - \mathbf{y}) = \mathbf{0}$	**M1**		**3**
	(Since $\mathbf{x} \neq \mathbf{y}$) $\lambda = \frac{1}{2}$	**A1**		
(ii)	$(X * Y) * Z = \lambda(\lambda\mathbf{x} + (1-\lambda)\mathbf{y}) + (1-\lambda)\mathbf{z}$	**M1**		
	$= \lambda\, 2\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)\mathbf{z}$	**A1**		
	and			
	$X * (Y * Z) = \lambda\mathbf{x} + (1-\lambda)[\lambda\mathbf{y} + (1-\lambda)\mathbf{z}]$	**M1**		
	$= \lambda\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)2\mathbf{z}$	**A1**		
	$(X * Y) * Z - X * (Y * Z) = \lambda(1-\lambda)(\mathbf{x} - \mathbf{z})$	**M1**		**6**
	The two are distinct provided $\lambda \neq 0, 1$ or $X \neq Z$	**A1**		
(ii)	$(X * Y) * Z = \lambda^2\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)\mathbf{z}$			
	$(X * Z) * (Y * Z) = [\lambda\mathbf{x} + (1-\lambda)\mathbf{z}] * [\lambda\mathbf{y} + (1-\lambda)\mathbf{z}]$	**M1**		
	$= \lambda^2\mathbf{x} + \lambda(1-\lambda)\mathbf{z} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)^2\mathbf{z}$			
	$= \lambda^2\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)\mathbf{z}$		**A1** (and the two are always equal)	

$X * (Y * Z) = \lambda\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)^2\mathbf{z}$

$(X * Y) * (X * Z) = [\lambda\mathbf{x} + (1-\lambda)\mathbf{y}] * [\lambda\mathbf{x} + (1-\lambda)\mathbf{z}]$ **M1**

$= \lambda^2\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + \lambda(1-\lambda)\mathbf{x} + (1-\lambda)^2\mathbf{z}$

$= \lambda^2\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)\mathbf{z}$ **A1**

Hence $X * (Y * Z) = (X * Y) * (X * Z)$ **E1** Conclusion must be stated (before or after proof)

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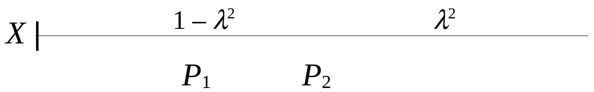
For $0 < \lambda < 1$, P_1 cuts XY in the ratio $(1-\lambda) : \lambda$ **B1**



M1 Iterating

We see that P_n cuts XY in the ratio

$$1 - \lambda^n : \lambda^n$$

A1A1 Any correct form

$P_{n+1} = P_n * Y = \lambda\,\mathbf{p}_n + (1-\lambda)\mathbf{y}$

$= \lambda\left\{(\lambda^n)\mathbf{x} + (1-\lambda^n)\mathbf{y}\right\} + (1-\lambda)\mathbf{y}$

$= \lambda^{n+1}\mathbf{x} + (\lambda - \lambda^{n+1} + 1 - \lambda)\mathbf{y}$

$= \lambda^{n+1}\mathbf{x} + (1-\lambda^{n+1})\mathbf{y}$

and proof follows by induction **M1A1** for attempt at an inductive proof; fully correct

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SI 2013 Mark Scheme Q4

(i)	$\int \tan^n x.\sec^2 x \, dx = \left[\frac{1}{n+1}\tan^{n+1} x\right]$	**M1**	May be done via a substn. such as $t = \tan x$ (or by "parts")	**2**
	$= \frac{1}{n+1}$	**A1**	**ANSWER GIVEN**	
	$\int \sec^n x.\tan x \, dx = \int \sec^{n-1} x.\sec x \tan x \, dx$	**M1**	May be done via a substn. such as $s = \sec x$ (or by parts)	
	$= \left[\frac{1}{n}\sec^n x\right]$	**A1**		
	$= \frac{\left(\sqrt{2}\right)^n - 1}{n}$	**A1**	**ANSWER GIVEN**	**3**

(ii) (a) $\int_0^{\pi/4} x\sec^4 x \tan x \, dx = \left[x.\frac{\sec^4 x}{4}\right]_0^{\pi/4} - \int_0^{\pi/4} \frac{\sec^4 x}{4} dx$ **M1A1A1** for appropriate use of *parts*; correct (1st, 2nd)

$= \frac{\pi}{4} - \frac{1}{4}J$ where $J = \int_0^{\pi/4} \sec^4 x \, dx$

$J = \int_0^{\pi/4} \sec^2 x \, dx + \int_0^{\pi/4} \sec^2 x \tan^2 x \, dx$ **M1** for use of $\sec^2 x = 1 + \tan^2 x$ to split up the integral

$= \left[\tan x + \tfrac{1}{3}\tan^3 x\right]$ **A1 A1**

$= \dfrac{4}{3}$ NB Limits are ignored until the end, when numerical answers need to appear

Thus $I = \dfrac{\pi}{4} - \dfrac{1}{3}$ **A1**

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(ii) (b) $\int x^2\left(\sec^2 x . \tan x\right) dx$ **M1** for appropriate splitting and attempted use of *integration by parts*

$= \left[x^2 . \tfrac{1}{2}\tan^2 x\right] - \int 2x . \tfrac{1}{2}\tan^2 x \, dx$ **A1A1**

$= \dfrac{\pi^2}{32} - \int x\left(\sec^2 x - 1\right)$ **M1** for use of $\tan^2 x = \sec^2 x - 1$

$= \dfrac{\pi^2}{32} - K + \int x \, dx$ where $K = \int_0^{\pi/4} x \sec^2 x \, dx$

$K = \left[x . \tan x\right] - \int \tan x \, dx$ **M1**

$= x \tan x - \ln(\sec x)$ **B1** for $\int \tan x \, dx = \ln(\sec x)$

$= \dfrac{\pi}{4} - \dfrac{1}{2}\ln 2$ **A1**

Thus $I = \dfrac{\pi^2}{32} - \left(\dfrac{\pi}{4} - \dfrac{1}{2}\ln 2\right) + \dfrac{\pi^2}{32}$

$= \dfrac{\pi^2}{16} - \dfrac{\pi}{4} + \dfrac{1}{2}\ln 2$ **A1**

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Note that there are many ways to split these integrals in (ii) into parts.

SI 2013 Mark Scheme Q5

(i)	For $k = 0$: $x^2 + 3x + y^2 + y = 0$			
	$\left(x + \frac{3}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \left(\frac{1}{2}\sqrt{10}\right)^2$	**M1**	for completing the square for both x and y	
	giving a CIRCLE	**G1**	for a circle drawn	
	thro' (0, 0), (0, –1) & (–3, 0)	**B1B1**	passing thro' the origin; other 2 intercepts correct (noted on sketch or separately)	**4**
(ii)	For $k = \frac{10}{3}$: $(3x + y)(x + 3y + 3) = 0$	**B1**	Must be the full thing (no marks for just factorising the given quadratic)	
	giving a LINE-PAIR	**G1**	for two (intersecting) lines drawn	
	Lines $y = -3x$ & $x + 3y = -3$		Statement of eqns. not actually required	
	1st line thro' O with $-_{ve}$ gradient	**B1**		
	2nd line not thro' O with $-_{ve}$ gradient	**M1**		
	thro' (0, –1) & (–3, 0)	**A1**	for both stated or noted on sketch	**5**
(iii)	For $k = 2$: $(x + y)^2 + 3x + y = 0$			
	When $\theta = 45^{\circ}$,			

$x + y = X\sqrt{2}$ and $y - x = Y\sqrt{2}$	**B1**	noted or used anywhere
$\Rightarrow$ $x = \dfrac{X - Y}{\sqrt{2}}$ and $y = \dfrac{X + Y}{\sqrt{2}}$	**M1A1**	
$(x + y)^2 + 3x + y = 0$ becomes …		
$2X^2 + \dfrac{3X - 3Y}{\sqrt{2}} + \dfrac{X + Y}{\sqrt{2}} = 0$	**M1**	for eliminating both x and y for X and Y
$\Rightarrow 2X^2 + 2\sqrt{2}X = Y\sqrt{2}$		
$\Rightarrow (\sqrt{2}X + 1)^2 - 1 = Y\sqrt{2}$	**M1A1**	for completing the square; correct **ANSWER GIVEN** obtained

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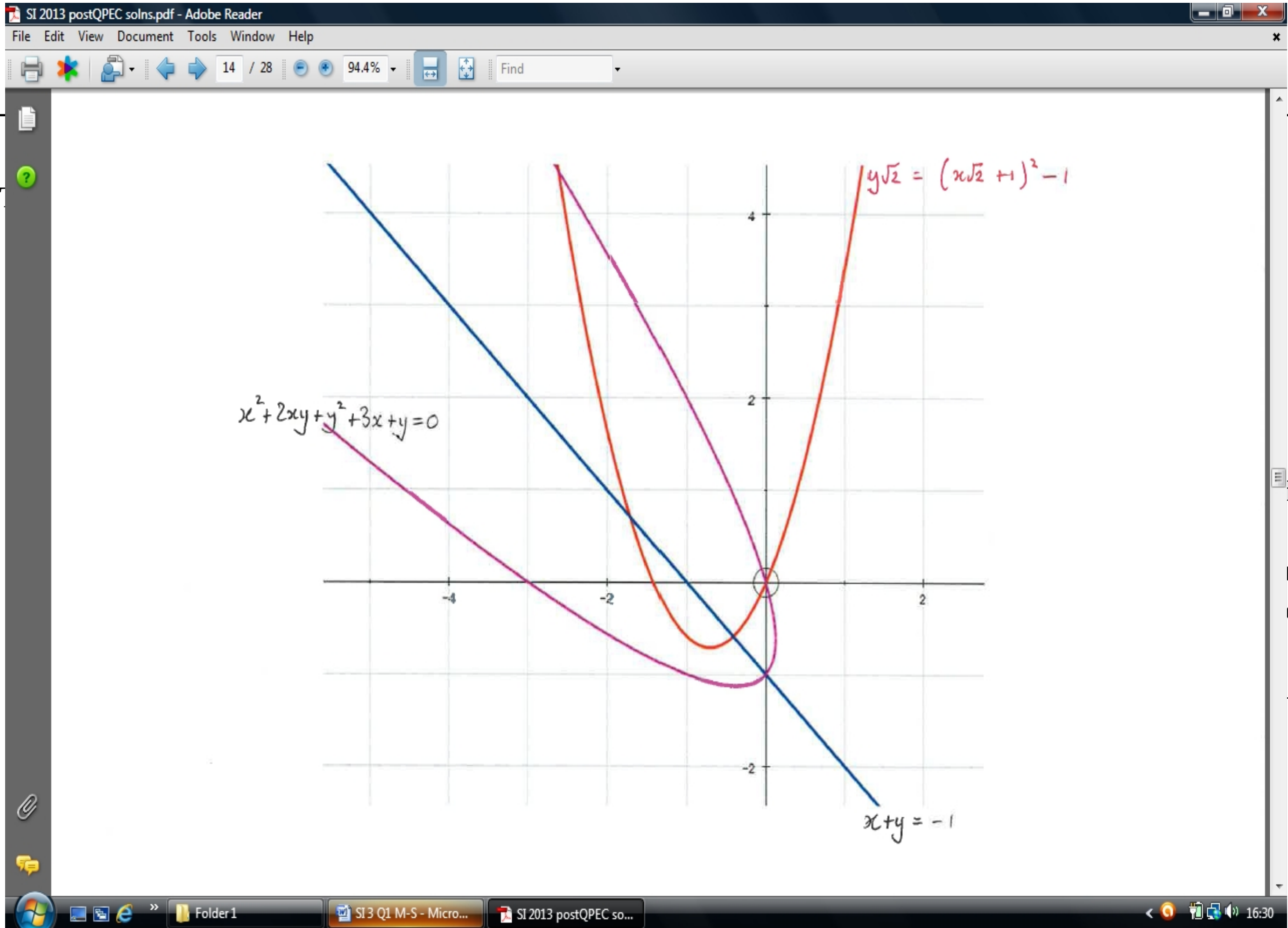

T

l curve correct

rple curve and blue line obviously
otation thro' 45° of their red curve

.L correct (incl. thro' O)

If red curve does not appear …
G3 for purple curve & blue line correct
G1 for obviously rotated 45° c/w
G0 for anything else

5

SI 2013 Mark Scheme Q6

(*) Coefft. of x^r in $(1 + x)^{n+1}$ is $\binom{n+1}{r}$ **B1**

Coefft. of x^r in $(1 + x)(1 + x)^n$ is from

$$(1+x)\left(\;.....+\binom{n}{r-1}x^{r-1}+\binom{n}{r}x^r+.....\; \right)$$ **M1**

$$\Rightarrow \binom{n+1}{r}=\binom{n}{r-1}+\binom{n}{r}$$ **A1** **GIVEN ANSWER** legitimately obtained

3

For n even, writing $n = 2m$ … **M1** for attempting the even case

$$B_{2m} + B_{2m+1} = \binom{2m}{0}+\binom{2m-1}{1}+\binom{2m-2}{2}+.....+\binom{m+1}{m-1}+\binom{m}{m}$$ **B1**

$$+\binom{2m+1}{0}+\binom{2m}{1}+\binom{2m-1}{2}+\binom{2m-2}{3}+.....+\binom{m+1}{m}$$ **B1**

$= \binom{2m+1}{0} + \left[\binom{2m}{0} + \binom{2m}{1}\right] + \left[\binom{2m-1}{1} + \binom{2m-1}{2}\right] + \ldots\ldots + \left[\binom{m+1}{m-1} + \binom{m+1}{m}\right] + \binom{m}{m}$ **M1** for suitable pairings (clear)

$= \binom{2m+2}{0} + \left[\binom{2m+1}{1}\right] + \left[\binom{2m}{2}\right] + \ldots\ldots + \left[\binom{m+2}{m}\right] + \binom{m+1}{m+1}$ **M1** for use of first result, **(∗)**

using the result **(∗)** from above and since $\binom{2m+1}{0} = \binom{2m+2}{0} = \binom{m}{m} = \binom{m+1}{m+1} = 1$ **M1** for noting the equality of the "1"s

$= \sum_{j=0}^{m+1} \binom{2(m+1)-j}{j} = B_{2m+2}$ **A1** Legitimately shown

7

For n odd, writing $n = 2m + 1$ … **M1** for attempting the odd case

$B_{2m+1} + B_{2m+2} = \binom{2m+1}{0} + \binom{2m}{1} + \binom{2m-1}{2} + \ldots\ldots + \binom{m+2}{m-1} + \binom{m+1}{m}$ Note that this appeared above

$+ \binom{2m+2}{0} + \binom{2m+1}{1} + \binom{2m}{2} + \binom{2m-1}{3} + \ldots\ldots + \binom{m+2}{m} + \binom{m+1}{m+1}$ **B1**

$= \binom{2m+2}{0} + \left[\binom{2m+1}{0} + \binom{2m+1}{1}\right] + \left[\binom{2m}{1} + \binom{2m}{2}\right] + \ldots\ldots + \left[\binom{m+2}{m-1} + \binom{m+2}{m}\right] + \left[\binom{m+1}{m} + \binom{m+1}{m+1}\right]$

M1 for suitable pairings (clear)

$= \binom{2m+3}{0} + \left[\binom{2m+2}{1}\right] + \left[\binom{2m+1}{2}\right] + \ldots\ldots + \left[\binom{m+3}{m}\right] + \binom{m+2}{m+1}$

M1 for use of first result, **(∗)**

using the result **(∗)** from above and since $\binom{2m+2}{0} = \binom{2m+3}{0} = 1$ **M1** for noting the equality of the "1"s

6

$$= \sum_{j=0}^{m+1} \binom{2(m+1)+1-j}{j} = B_{2m+3}$$ **A1** Legitimately shown

B_0 , B_1 = $\binom{0}{0}, \binom{1}{0} = 1,\ 1$ **B1** Evaluating B_0 , B_1

Evaluating F_0 , F_1 & $F2$ **B1** Both sides must be evaluated, not just stated

Statement that $B_n = F_{n+1}$ **B1** At any point

For a clear justification of result (e.g. inductively) **E1**

(i.e. comment that since $B_0 = F_1$, $B_1 = F_2$ and B_n & F_n satisfy the same recurrence relation, we must have $B_n = F_{n+1}$ for all n)

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SI 2013 Mark Scheme Q7

(i) $y = ux \Rightarrow \dfrac{dy}{dx} = u + x\dfrac{du}{dx}$ **B1**

$\Rightarrow u + x\dfrac{du}{dx} = \dfrac{1}{u} + u$ **M1** substituted in

$\Rightarrow \int u\,du = \int \dfrac{1}{x}\,dx$ **M1** variables separated and integration attempted

$\Rightarrow \tfrac{1}{2}u^2 = \dfrac{y^2}{2x^2} = \ln x \quad (+C)$ **M1** substituted back for x and y

$\Rightarrow y^2 = x^2(2\ln x + 2C)$

$x = 1$, $y = 2$ substd. to determine C **M1** $C = 2$ (the lack of a constant of integration earlier $\Rightarrow$ M0 here)

$\Rightarrow y = x\sqrt{2\ln x + 4}$ **ANSWER GIVEN** (all working and signs must be correct throughout)

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		E1	Justification of $+_{ve}$ square-root required: $y > 0$ when $x = 1$ gives this (Given $x > e^{-2}$, so square-rooting valid – this does not need to be stated by candidates)
(ii)	$y = ux$ used to get $u + x\dfrac{du}{dx} = \dfrac{1}{u} + 2u$	**B1**	
	$\Rightarrow \displaystyle\int \frac{2u}{1+u^2}\,du = \int \frac{2}{x}\,dx$	**M1**	variables separated and integration attempted
	$\Rightarrow \ln(1+u^2) = 2\ln x \quad (+\ln A)$	**A1**	
	$x = 1,\ y = 2$ substd. to determine A	**M1**	$A = 5$ (the lack of a constant of integration earlier $\Rightarrow$ M0 here)
	$1 + \dfrac{y^2}{x^2} = 5x^2$	**M1**	for eliminating u and correct use of log laws (*with* correct number of terms)
	$y = x\sqrt{5x^2 - 1}$	**A1**	No need to justify choice of $+_{ve}$ square-root, but don't allow ±
	for $x > \frac{1}{\sqrt{5}}$	**B1**	**FT** sensible domains (provided $x > 0$)

(ii) ALTERNATIVE

	$y = ux^2$ used to get $2ux + x^2\dfrac{du}{dx} = \dfrac{1}{ux} + 2ux$	**B1M1**	$\dfrac{dy}{dx}$ correct (LHS); full substitution into given d.e.
	$\Rightarrow \displaystyle\int u\,du = \int \frac{1}{x^3}\,dx$	**M1**	variables separated and integration attempted
	$\Rightarrow \frac{1}{2}u^2 = \dfrac{-1}{2x^2} \quad (+\frac{1}{2}C)$		
	$x = 1,\ y = 2\ (u = 2)$ substd. to find C	**M1**	C = l5 (the lack of a constant of integration earlier $\Rightarrow$ M0 here)
	$\dfrac{y^2}{x^4} = 5 - \dfrac{1}{x^2}$	**M1**	for eliminating u
	$y = x\sqrt{5x^2 - 1}$	**A1**	No need to justify choice of $+_{ve}$ square-root, but don't allow ±
	for $x > \frac{1}{\sqrt{5}}$	**B1**	**FT** sensible domains (provided $x > 0$)

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(iii)	$y = ux^2$ used to get $2ux + x^2\dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{u} + 2ux$	**B1M1**	$\dfrac{\mathrm{d}y}{\mathrm{d}x}$ correct (LHS); full substitution into given d.e.
	$\Rightarrow \int u\,\mathrm{d}u = \int \dfrac{1}{x^2}\,\mathrm{d}x$	**M1**	variables separated and integration attempted
	$\Rightarrow \frac{1}{2}u^2 = \dfrac{-1}{x}\ (+D)$		
	$x = 1,\ y = 2\ (u = 2)$ substd. to find D	**M1**	$D = 3$ (the lack of a constant of integration earlier $\Rightarrow$ M0 here)
	$\dfrac{y^2}{x^4} = 6 - \dfrac{2}{x}$	**M1**	for eliminating u
	$y = x\sqrt{6x^2 - 2x}$ or $y = x\sqrt{2x}\sqrt{3x-1}$	**A1**	No need to justify choice of $+_{\text{ve}}$ square-root, but don't allow $\pm$
	for $x > \frac{1}{3}$	**B1**	**FT** sensible domains (provided $x > 0$)

7

SI 2013 Mark Scheme Q8

	function	domain	range		
(i)	$\mathrm{cb}(x) = 2\ln x$	$x > 0$	$\mathbb{R}$	**B1**	
	$\mathrm{ab}(x) = (\ln x)^2$	$x > 0$	$y \geq 0$	**B1**	
	$\mathrm{da}(x) = \lvert x \rvert$	$\mathbb{R}$	$y \geq 0$	**B1**	Do not allow x or $\sqrt{x^2}$ here; but $\begin{cases} x & x > 0 \\ -x & x < 0 \end{cases}$ is ok
	$\mathrm{ad}(x) = x$	$x \geq 0$	$y \geq 0$	**B1**	
					These B1s are for the functions.
				B1B1	1st for all domains correct; 2nd for all ranges correct

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(ii) fg(x) = | x | or $\sqrt{x^2}$ $\quad\mathbb{R}\quad$ $y \geq 0$ **B1 B1** Allow $\begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$ but not or $\pm x$

gf(x) = | x | or $\sqrt{x^2}$ $\quad |x| \geq 1\quad$ $y \geq 1$ **B1 B1**

In each case, 1st B1 for the fn. & 2nd B1 for both domain and range
OR 1st B1 for both domains correct, 2nd B1 for both ranges correct

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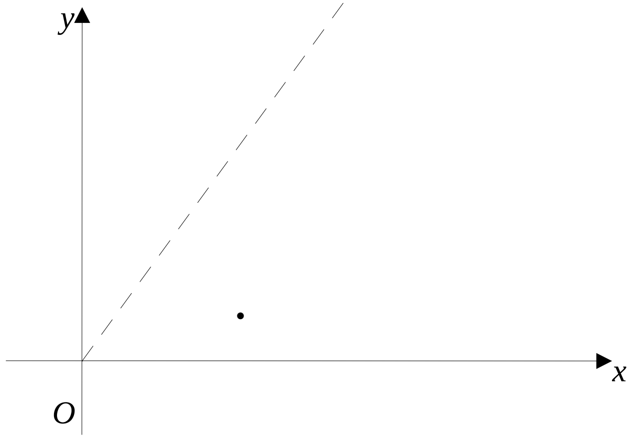
(iii) $y = \text{h}(x) = x + \sqrt{x^2 - 1}$, $x \geq 1$



G1 Starts at (1, 1) – noted or clear on sketch

G1 Going upwards $(\to \infty)$

G1 Approaching asymptote $y = 2x$ (eqn. must be given somewhere)

NB – I will correct the diagram so it is the graph of a *function*

$y = \text{k}(x) = x - \sqrt{x^2 - 1}$, $|x| \geq 1$

y

G1 Two branches

G1 (1, 1) and (–1, –1) noted or clear on sketch

G1 Branch in Q1 approaches (from above) asymptote $y = 0$ (or "x-axis") must be noted somewhere

x

O

G1 Branch in Q3 approaches asymptote (from above) $y = 2x$ must be noted somewhere

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Domain of kh is $x \geq 1$	**B1**	
Range of h is $y \geq 1 \Rightarrow$ Range of kh is $0 < y \leq 1$	**M1A1**	**3**