STEP CORRESPONDENCE PROJECT

Postmortem: Assignment 12

Warm-up

1 **(i)** We were not looking for anything fancy here. $n(n+1)$ is the product of two consecutive integers, so one of them is divisible by 2. Hence $n(n+1)$ is divisible by 2.

$n(n+1)(n+2)$ is the product of three consecutive integers, so one of them is a multiple of 3 and one (at least) is divisible by 2. The product is therefore divisible by 6.

(ii) $n^3 - n = (n-1)n(n+1)$, and so is the product of three consecutive integers. Therefore it is divisible by 6, as above.

(iii) $n^5 - n^3 = n^3(n-1)(n+1)$. From part **(ii)** it is divisible by three. Then if n is even, n^3 is divisible by 8, and if n is odd, $n-1$ and $n+1$ are consecutive even numbers — which means one of them is a multiple of 4 and the other is a multiple of 2.

(iv) $2^{2n} - 1 = (2^n - 1)(2^n + 1)$. Note that you cannot say "this is the product of two consecutive odd numbers, so one of them must be divisible by 3" — consider 5 and 7!

Instead consider $(2^n - 1)2^n(2^n + 1)$ which is the product of three consecutive integers, so must be divisible by 3. But 2^n is **not** divisible by 3.

(v) Put $n = 3k + 1$ into $n^3 - 1$ and expand.

Preparation

2 **(i)** Note you can cancel out some ms in the second batch of fractions in part **(b)** first.

(ii) Remember that the probabilities for the second sweet depend upon what happened for the first sweet (they are **conditional**).

(iii) Again, the probabilities change depending on what happened with the first sweet. If the first was an apple sour, and you remove it, you are then left with $a-1$ apple sours and a total of $a+b-1$ sweets.

(iv) This question was mainly an exercise in reading the question carefully.

The STEP question

3 **(i)** As long as you have a £1 coin person first, then you can give change to the one £2 coin person whenever they arrive.

(ii) Here the options that work are (omitting the pound signs):

- 1, 1, anything
- 1, 2, 1.

Note that you don't have to consider (1, 1, 1) and (1, 1, 2) separately. You can of course consider the options that **don't** work instead.

(iii) Here the options that work are:

- 1, 1, 1, anything
- 1, 1, 2, 1, anything
- 1, 1, 2, 2, 1
- 1, 2, 1, 1, anything
- 1, 2, 1, 2, 1.

You need to be sure you have covered all the possible options, and explain in your proof why this is so. Working systematically helps.

This was a question from the Probability and Statistics section of a STEP paper, and is the first non-pure question we have used. Note that it doesn't actually need anything other than GCSE probability and algebraic fraction manipulation, but does require you to read the question carefully, work logically and have fluent algebraic manipulation skills.

Warm down

4 Start by calling the person in part **(ii)** Xander (X), the person in part **(iii)** Yolanda (Y), and the person in part **(iv)** Zoe (Z). You can then put all the information from the first 5 parts of the question in a table:

Race	1st	2nd	3rd	4th	5th
There	Y		Z	A	X
Back	X	Z	Y	B	

You now know that Anna and Bryan are **not** any of X, Y and Z. The missing space for the race to the tree must be Bryan and the missing space on the race back must be Anna. Then use the information from the last two parts to work out which of X, Y and Z must be Charlie.