STEP CORRESPONDENCE PROJECT

Postmortem: Assignment 10

Warm-up

1 **(i)** Do be careful with brackets. For example, write

$$\cos\alpha\cos\beta \times (\tan\alpha + \tan\beta)$$

(if this is what you mean) rather than

$$\cos\alpha\cos\beta \times \tan\alpha + \tan\beta.$$

You can find $\sin(\alpha - \beta)$ from $\sin(\alpha + (-\beta))$ using the previous result.

You can find $\cos(\alpha + \beta)$ from $\cos(\alpha + \beta) = \sin(90^\circ - (\alpha + \beta)) = \sin((90^\circ - \alpha) - \beta)$.

Note that when you have the expression for $\sin(\alpha + \beta)$, you can derive $\sin(\alpha - \beta)$ and $\cos(\alpha \pm \beta)$, and $\tan(\alpha \pm \beta)$ with hardly any extra work.

(ii) $\sin 75^\circ = \sin(45^\circ + 30^\circ)$ and $\sin 15^\circ = \sin(45^\circ - 30^\circ)$. At the risk of repeating ourselves: when a question asks for **values** then decimal approximations are not the required form.

(iii) You will need expressions for $\sin 2A$ and $\cos 2A$, but these can be found from the results in part **(i)**, for example $\sin 2A = \sin(A + A) = \cdots$. You will probably also need to use $\sin^2 A + \cos^2 A = 1$.

Preparation

2 **(ii)** Note that if a is a *positive* integer then $a - 1$ is an integer satisfying $a - 1 \geqslant 0$.

(iv) These sorts of statements can be a bit confusing. I usually re-order them in my head, so

"$a = b$ **if** $a^2 = b^2$" becomes "**if** $a^2 = b^2$ **then** $a = b$" which I can now see is false

and

"$a = b$ **only if** $a^2 = b^2$" becomes "**only if** $a^2 = b^2$ **is it the case that** $a = b$" which I can now see is true.

(v) **(d)** This statement is not an **iff**. It is certainly the case that the triangle is right-angled **if** $a^2 + b^2 = c^2$, but this is not a necessary condition (it is not **only if**): the triangle is also right-angled if $a^2 = b^2 + c^2$ (i.e. a is the hypotenuse).

The STEP question

3 **(i)** **(b)** If you use the hint provided and write $N = p_1^{a_1} \times p_2^{a_2} \times \cdots$ then you should find that f(N) is the product of several things all of the form $p^a \left(1 - \frac{1}{p}\right)$. You should then be able to show that each of these in an integer (and hence that the whole product is an integer).

(ii) **(a)** A simple counterexample (remember the simpler the better!) would be $\mathrm{f}(12) \neq \mathrm{f}(2) \times \mathrm{f}(6)$. You show it is indeed a counterexample by calculating the three values of the function.

(b) This result is true, and can be shown just by using the definitions such as $\mathrm{f}(p) = p\left(1 - \frac{1}{p}\right)$. When p and q are not distinct ($p = q$), the result is not true.

(c) Try f(4) and f(15). The statement $\mathrm{f}(p)\mathrm{f}(q) = \mathrm{f}(pq)$ holds as long as p and q are *co-prime*, that is, they have no prime factors in common.

When p and q have a prime factor in common then $\mathrm{f}(pq) \neq \mathrm{f}(p)\mathrm{f}(q)$

(iii) Start by writing $\mathrm{f}(p^m) = p^m\left(1 - \frac{1}{p}\right) = p^{m-1}(p-1)$. Then express 146410 as a product and compare this to $p^{m-1}(p-1)$ and you should be able to see what p and m are.

Warm down

4 **(i)** Leave answers in fractions (unless specifically asked for a decimal approximation).

(ii) There is nothing wrong with the negative solution (nothing in the question says that the numbers have to be positive) so put down both solutions.

(iii) Two of the terms simplify nicely, but the other two do not.

(iv) Most people knew to multiply numerator and denominator by $\sqrt{3} - 1$, but simplfying $\sqrt{12 + 6\sqrt{3}} \times (\sqrt{3} - 1)$ proved to be a bit more complicated.

One way to proceed it to take the $\sqrt{3} - 1$ inside the square root, so then you have to simplify $\sqrt{(12 + 6\sqrt{3}) \times (\sqrt{3} - 1)^2}$.