STEP CORRESPONDENCE PROJECT

Postmortem: Assignment 11

Warm-up

1 **(i)** You should have recognised this sequence (and the fact that it is called T is a hint!).

(ii) This function is a periodic function. You should find that it looks a bit like saw teeth.

Preparation

2 **(i)** You should note that $4^x = (2^2)^x = 2^{2x} = (2^x)^2 = y^2$, so you need to factorise $y^2 - 7y - 8 = 0$. Since $y = 2^x$ you know that y must be positive.

(ii) You need to be very careful if you use squaring when solving an equation. It often means that extra "solutions" are created. As a very simple example, the equation:

$$x + 1 = 2$$

has just the one solution, $x = 1$. However, if our first step was to square both sides, we would get:

$$\begin{aligned} (x+1)^2 &= 4 \\ x^2 + 2x + 1 &= 4 \\ x^2 + 2x - 3 &= 0 \\ (x+3)(x-1) &= 0 \\ x &= -3, x = 1\,. \end{aligned}$$

If you square both sides of an equation, you must make sure that you check your solutions in the **original** equation to make sure that they are indeed solutions.

Note that $\left(\sqrt{A} - \sqrt{B}\right)^2 \neq A - B$ or even $A + B$. When you square both sides (which you may do before or after taking $\sqrt{x+6}$ to the other side) you will get a cross term which is still a square root. However, you now only have one square root term, so isolate this on one side of the equation and square again.

The STEP question

3 Substitutions work quite nicely for all three equations. In the first case you should end up with $y = \dfrac{-3 \pm \sqrt{11}}{2}$. Then you should note that as $y = \sqrt{x}$, only the positive solution makes sense, and $x = \left(\dfrac{-3 + \sqrt{11}}{2}\right)^2 = \cdots$

The other two parts follow in the same way, there is one root for part **(ii) (a)** and two roots for part **(ii) (b)**. Using substitutions is probably the simplest method of solving them, remembering to reject solutions where y is negative. The obvious substitutions are $y = \sqrt{x+2}$ for part **(ii) (a)** and $y = \sqrt{2x^2 - 8x - 3}$ for part **(ii) (b)**.

Squaring can be used (but is not necessary which carefully chose substitutions). If you take this approach then you must check your solutions in the original equation.

Warm down

4 **(i)** As at least one person has pointed out, if m and n are allowed to be any real numbers then there are infinitely many solutions. For any n we can find m by using $m = 3 + (n-1)\log_2(3)$. The question was meant to say that m and n are integers, in which case there is only 1 possible value for each of m, n. The key point here is that if $2^x = 3^y$ where x, y are both integers then we must have $x = y = 0$.

(ii) The trick here is to use two substitutions, such as $3^x = a$ and $5^x = b$. You will then end up with the equation $15a^2 - 34ab + 15b^2 = 0$. You can then find two possible relationships between a and b, which can be used to find two possible values of x.