STEP CORRESPONDENCE PROJECT

Postmortem: Assignment 15

Warm-up

1 **(i)** You can replace b with $-b$ to get the second result (or if you are uncomfortable with this you can let $b = -c$).

(ii) **(b)** You may get slightly different answers depending on whether your first step is

$$(x^3)^2 - (y^3)^2 = (x^3 + y^3)(x^3 - y^3)$$

(difference of two squares) or

$$(x^2)^3 - (y^2)^3 = (x^2 - y^2)(x^4 + x^2y^2 + y^4)$$

(using the result of question 1).

(iii) After finding the sum of the geometric progression, use the substitution $t = \frac{b}{a}$. For $a^5 + b^5$ you can replace b with $-b$ as in question 1.

(iv) Multiplying out the brackets gives $x^4 + y^4$.

Then, taking $y = 1$, you get

$$x^4 + 1 = (x^2 + 1 - \sqrt{2}\,x)(x^2 + 1 + \sqrt{2}\,x).$$

Setting this equal to zero gives two quadratic equations which you can solve to find the four fourth roots of -1. Clearly, they will be complex (not real), since $x^4 \geqslant 0$ if x is real.

Preparation

2 **(i)** You should get $\frac{3}{2}$.

(ii) This is a GP of the form $\sum r^k$, so for convergence you need to show that $|r| < 1$. which you can do by noting (or proving) that $\sqrt{3} < 2$. The sum to infinity is $3(2 + \sqrt{3})$.

(iii) After expanding (binomial series, if you like) and simplifying $(2 + \sqrt{3})^5$ you can get the second result from just changing the sign of $\sqrt{3}$ (which can be done in your final simplified version) — compare with question 1. Note the use of the instruction *Write down* which is supposed to indicate that no extra work is required.

(iv) By factorising as suggested you get $(x-y)(x+y)=z$ i.e. $z(x+y)=z$. This means that either $z=0$ or $x+y=1$. If is important that you consider the case $z=0$ separately and don't just divide by z.

It turns out that $x^2=-2$ when $z=0$, so there are no real solutions in this case. In fact, the question does not ask for *real* solutions, so you should give this one as well ($x=\pm i\sqrt{2}$, $y=\ldots$).

The STEP question

3 **(i)** For $n=0$, $F_0=0$ so $a+b=0$. Thus $b=-a$ and $F_n=a(\lambda^n-\mu^n)$. The other equations ($n=1$, 2, 3) are:

$$\begin{aligned} a(\lambda-\mu)&=1\\ a(\lambda^2-\mu^2)&=1\\ a(\lambda^3-\mu^3)&=2. \end{aligned}$$

You can then use the result from question **1(i)** to get the required result (noting that $\lambda\neq\mu$!). You can also use the first two equations to get $\lambda+\mu=1$.

When solving the equations, it is helpful to notice that as $a>0$ and $a(\lambda-\mu)=1$ then $\lambda>\mu$.

(ii) If you noticed that the sequence is the Fibonacci sequence, then you should easily calculate that $F_6=8$. However you have to show this by using $F_n=a(\lambda^n-\mu^n)$, and as the answer is known (or easily found by other means) you must be careful to fully justify your answer.

(iii) Note that the sum can be written as $\displaystyle\sum_{n=0}^{\infty}\frac{a(\lambda^n-\mu^n)}{2^{n+1}}$ or $\displaystyle\frac{a}{2}\sum_{n=0}^{\infty}\left(\frac{\lambda}{2}\right)^n-\frac{a}{2}\sum_{n=0}^{\infty}\left(\frac{\mu}{2}\right)^n$. It is probably easier to find the infinite sums in terms of a, λ and μ before substituting for them. You should get a nice answer.

Warm down

4 If we start on the first day with:

$$\begin{aligned} &\text{Group 1}: ABC\\ &\text{Group 2}: DEF\\ &\text{Group 3}: GHI \end{aligned}$$

then on subsequent days A, B and C must be in seperate groups, say A in group 1, B in group 2 and C in group 3. D must then go with A, B and C in turn etc.