STEP CORRESPONDENCE PROJECT

Postmortem: Assignment 17

Warm-up

1 **(i)** Set $y = x^2$ on both sides of the equation to obtain $\text{f}(y)$ in terms of y. This defines f (it doesn't matter whether we use x or y to define the function). Since $y = x^2$ we must have $y \geqslant 0$, so the behaviour of $\text{f}(y)$ for $y < 0$ is not determined.

(ii) This time, set $y = \sqrt{x}$. Again we must have $y \geqslant 0$ by definition of the square root function.

(iii) Integrating gives $\text{f}(x) = x^3 + c$ and we can use the given value of $\text{f}(0)$ to find c.

(iv) Set $y = x^2$ and then proceed as in part **(iii)**.

(v) If we have $\text{f}(x) = a^x$ then $\text{f}(x + y) = a^{x+y}$. You then need to use $\text{f}(x) = a^x$ to find $\text{f}(x)\text{f}(y)$ and show that this is the same as $\text{f}(x + y)$. To find a you need to use the condition $\text{f}(1) = 2$ in $\text{f}(x) = a^x$.

(vi) This function appears to work in the opposite way from the one in part (v), so think about using the inverse to a^x, i.e. $\log_a(x)$. You then need to show that your function does indeed satisfy the given relationship.

(vii) A function of the form $\text{f}(x) = Ax$ will work where A is a constant which can be found by using $\text{f}(2) = 1$.

Preparation

2 **(i)** There are lots of ways you can do these (e.g. $\sin 105^\circ = \sin(60^\circ + 45^\circ)$). There is nothing wrong with using your calculator to check your solution approximately, but only **after** working it exactly out yourself.

(ii) You should end up with a cubic in $\cos A$. You might find it easier to manipulate if you use the substitution $c = \cos A$. One solution is $c = 1$ (which gives $A = 0$ only, since 2π is out of range). There are 5 possible values of A.

(iii) We suggested that to show that $1+\sqrt{2}$ is a root you can substitute it and and show that does indeed give 0. When do the substitution, you see that each term has a common factor of $1+\sqrt{2}$ which you can factorise out.

You can also show that $x = 1+\sqrt{2}$ is a root by showing that $x-1-\sqrt{2}$ is a factor by algebraic division (which, as you are going to have to do this anyway to find the other roots, is not a totally bad idea). However, if you are going to show that $x = 1+\sqrt{2}$ is a root by this method then you need to show the long division very carefully and show that the last subtraction does give zero explicitly. You also need to state your final conclusion such as "hence $1+\sqrt{2}$ is a root".

(iv) If we take $x = 3y$ then the first equation becomes the second, so the roots of the second equation are given by $y = \frac{1}{3}x$.

The STEP question

3 **(i)** Use the identities given in question **2 (i)**.

(ii) Using the same idea as in question **2 (ii)**, write $\cos 3\alpha$ in terms of $\cos\alpha$. It should then be easy to demonstrate that when $x = \cos\alpha$, $4x^3 - 3x - \cos 3\alpha$ is equal to 0.

You then need to divide by $x - \cos\alpha$, which needs careful long division. You get a quadratic in x which you can solve using the quadratic formula. The roots should simplify to something of the form $a\cos\alpha \pm b\sin\alpha$.

(iii) If you set $y = 2x$ the equation will reduce to something of the form in part **(ii)** with $\cos 3\alpha = \frac{1}{\sqrt{2}}$. This means that $\alpha = 15^\circ$ and you can use the work from part **(ii)** to find the values of x. Then you can find the values of y from $y = 2x$.

Warm down

4 **(i)** Use ASA to show that the triangles are congruent.

(ii) Here use SAS to show that $\triangle GCD$ is congruent to $\triangle GBD$.

(iii) We have just shown $GC = CB$, and from part **(i)** we know $GE = GF$. It follows by Pythagoras' theorem that $EC = FB$.

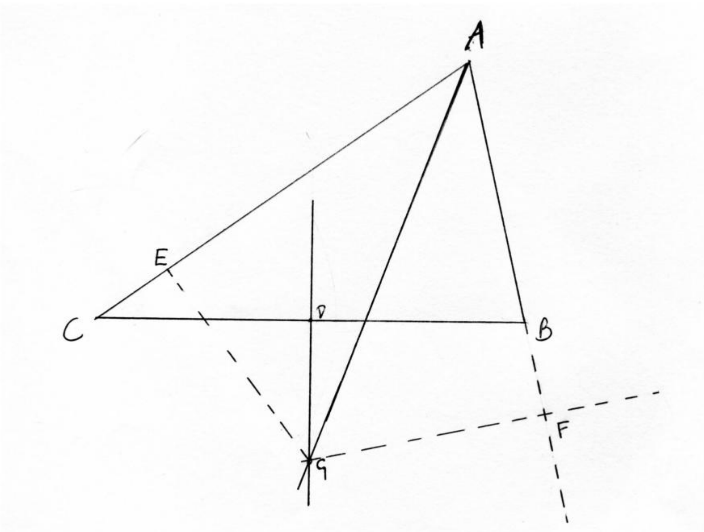
Hence as $AE = AF$ and $EC = FB$ we have $AC = AB$ and the triangle is isosceles.

(iv) It cannot be the case that **all** triangles are isosceles, so something is wrong with the "proof". Clearly, there is nothing wrong with the argument, so there must be something wrong with the diagram.

If you draw a non-isosceles triangle and carefully construct the points and lines as shown you should find that the point G lies outside of the triangle [1]. This is not a problem in itself, because the proof still works.

However, you should also find that exactly one of E and F lies outside the triangle, i.e. on either AC produced or AB produced. In the picture below, we have $AE = AF$ and $EC = FB$ **but** $AC = AE + EC$ and $AB = AF - FB$, so the proof now fails.

Please note that, although I have rubbed out the construction lines for clarity, they were there!



[1]You can prove that this must be the case. Assume that $AB < AC$ and let the angle bisector at A bisect the line BC at X. You can use the Sine Rule to show that $CX > XB$ and hence that the angle bisector at A intersects the perpendicular bisector to BC outside the triangle.