STEP CORRESPONDENCE PROJECT

Postmortem: Assignment 25

Warm-up

1 **(i)** For some, $\displaystyle\int \frac{u+2}{u}\,\mathrm{d}u$ was a sticking point. The thing to do it to split up the integrand, so that you integrate $1+\dfrac{2}{u}$.

(ii) If all goes well, you should find that the substitution leads to:

$$\int_0^{\frac{\pi}{6}} 1\,\mathrm{d}\theta\,.$$

Preparation

2 **(i)** Since you are given A and B in the question you should really stick with these variables (rather than using a and b). Use the compound angle formulae in the numerator and denominator and then divide throughout by $\cos A \times \cos B$.

(ii) You should end up with $-\ln\left(\frac{1}{2}+x\right)$, though there are a number of equally simple ways of writing this.

(iii) Divide throughout by $\cos^2\theta$.

(iv) We have not covered the derivation of the quotient rule (but that shouldn't stop you using it if you know it). The quotient rule (using a prime to denote differentiation)

$$\left(\frac{\mathrm{u}}{\mathrm{v}}\right)' = \frac{\mathrm{u'v} - \mathrm{v'u}}{\mathrm{v}^2}$$

follows immediately from the product rule applied to uv^{-1} using the chain rule to differentiate v^{-1} (which gives $-\mathrm{v}^{-2}\mathrm{v}'$).

(v) As we said, there are many approaches. You will need to re-write $\tan^2\alpha + 2\tan\alpha + 1$ as $(\tan\alpha + 1)^2$ at some point.

(vi) You need to make sure that you show a least one step (and preferably more than one) between:

$$I = \int_0^{84} \frac{x^2}{x^2+(84-x)^2}\,\mathrm{d}x$$

and

$$I = \int_0^{84} \frac{(84-u)^2}{u^2+(84-u)^2}\,\mathrm{d}u$$

since the answer is given!

The STEP question

3 After substituting and a bit of fiddling about you should have:

$$I = \int_0^{\frac{1}{4}\pi} \Big(\ln 2 - \ln(1+\tan\phi) \Big)\, \mathrm{d}\phi$$

which can be re-written as:

$$I = \left(\int_0^{\frac{1}{4}\pi} \ln 2 \,\mathrm{d}\phi \right) - I\,.$$

Don't forget that $\ln 2$ is a constant so when we integrate we get $\phi \times \ln 2$.

(i) Use $x = \tan\theta$.

(ii) You can get a hint of what to use by looking at the upper limit of the integral. Start with $x = 2u$ or similar.

Warm down

4 **(i)** For the last part, you'll need to remember that $\sin x = \cos(\frac{1}{2}\pi - x)$.

(ii) The result $\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$ is well worth memorising.

(a) Here $\mathrm{f}(x) = \sin x$.

(b) Write the integral in the form $\displaystyle\int \frac{\frac{1}{x}}{\ln x}\mathrm{d}x$.

(c) $S + T = x + c_1$ and $S - T = \ln(\cos x + \sin x) + c_2$. You can then use these two to find S and T in terms of x.