STEP CORRESPONDENCE PROJECT

Postmortem: Assignment 6

Warm-up

1 **(i)** Not much to say here **but** it is far, far easier to cancel the fractions before doing any multiplication (in fact if you cancel completely no multiplication is necessary).

The second part needs you to cancel out all the common factors and appreciate which ones are left — the first part should act as a guide. This is where those who multiplied out the fractions in the first part came unstuck.

Note: You should make sure your cancellations do not obliterate the underlying number — you might like to cancel in pencil.

Whilst working with some special cases if you are stuck is a good idea, you also need to show how the general case works (and not just by generalising from the special cases). This requires something like:

$$\frac{\frac{3}{2} \times \frac{5}{4} \times \cdots \times \frac{2n-1}{2(n-1)} \times \frac{2n+1}{2n}}{\frac{1}{2} \times \frac{3}{4} \times \frac{5}{8} \times \cdots \times \frac{2n-1}{2n}}$$

and then you can show everything cancels apart from two terms.

(ii) Some people used matrices to solve these equations. However that was not the easiest method (so don't worry if you have never heard of matrices), and mistakes were made in finding the inverse matrix.

I personally can never remember the inverse of a 3 by 3 matrix in terms of co-factors etc — I tend to Gauss-Jordan elimination if I have to find an inverse (and this method has the advantage that it can be used with any square matrix).

However, I always consider the alternatives before messing about with matrices as there is far too much room for error for my liking.

The easiest way for the first set of equations is to add pairs of equations, in each case two unknowns will cancel out and you can find the third one. It is quick to check whether your three values satisfy the three original equations, so do it!

For the second set of equations, adding the second and third equations gives the value of c, adding the first and second will give you a (which will be in terms of k) and then you can substitute a and c into whichever equation you fancy to find b (which is also in terms of k). When you have a in terms of k you should be able to see that $k = -1$ will be problematic (and if you add the first two equations you get $0 = 2$ which shows that in this case there is no solution to the equations).

Preparation

2 Some of you may have met permutations before, and some may have not so we assumed that it would be new to everybody. In general, if you have n objects where r_1 are identical, another r_2 are identical (but different to the first lot), and so on until the last r_k identical objects then there are:

$$\frac{n!}{r_1! \times r_2! \times \cdots \times r_k!}$$

distinct ways of arranging the objects.

Most people answered the questions correctly, but some of the explanations for part (iv) were a bit thin. If a question asks you to "show carefully", or if the answer is given, then you must fully explain your working (leave no gaps for the examiner to fill in). The best way to start is by considering the letters $L_1, I_1, L_2, L_3, I_2, A, N$.

We were happy for you to leave answers in terms of factorials here, but do be aware that A-levels may require you to find the actual number (it is probably safest to do both unless told otherwise).

The STEP question

3 **(i)** As the answer is given, you need to show carefully that you have all the possible numbers and not just stop when you get to 15 — you need to show there are no more. The key is to be systematic, and it is probably easiest to consider the different cases that arise according to the numbers of nines.

With the number 99999 the digit sum is 45, so this is out. However we can have 4 nines and a seven. There are 5 places in which we can put the seven, so 5 different numbers here.

With 3 nines, the other two digits need to sum to 16 so they must be 2 eights. You can use $\dfrac{5!}{3! \times 2!}$ to show that there are 10 different numbers with 3 nines and 2 eights.

If we then consider 2 nines then the highest digit sum we can make is $9+9+8+8+8 = 42$ so 2 nines is not possible. Similarly for 1 nine and 0 nines.

(ii) Again, start with 4 nines (and then you need a three). Then consider 3 nines, so the two remaining digits must sum to 12 so can be 2 sixes, a seven and a five or an eight and a four (be careful not to double count and include nine and three — this has already been covered in the 4 nines case).

You can then consider 2 nines, 1 nine and 0 nines, ending up with 10 different combinations of digits which sum to 39.

For each different combination of digits you can work out how many different numbers you can make using these digits, using the techniques practised in question 2.

Warm down

4 **(i)** There were quite a few people quoting Bayes' Theorem (which has come in and out of different modules over the years):

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

So the probability that a person is a woman given that they are a smoker is given by:

$$P(W|S) = \frac{P(\text{Woman and a Smoker})}{P(\text{Smoker})} = \frac{0.18}{0.38}$$

However, you did not need to know Bayes' Theorem formally for this question (in fact we were assuming that you didn't know it).

If you assume a population of 100 people you find that out of these 100, 38 are smokers and of these smokers 18 are women (i.e. there are 18 women smokers). Then the probability if you are considering the 38 smokers that you pick is women is therefore $\frac{18}{38}$.

(ii) This seems to be a very surprising result, given that the test appears to be very accurate. If you test positive you still only have a less than 5% chance of having the disease, which can be shown by either using Bayes' theorem or by considering a population as in part i.