STEP CORRESPONDENCE PROJECT

Postmortem: Assignment 9

Warm-up

1 **(i)** When showing that two triangles are congruent, you must fully justify your statements. For example, to justify $AB = BC$, you might write:

$AB = BC$ (given).

In the case of triangles BMA and BMC, you might write:

BM is common to both triangles.

Don't assume things that are not given, especially not if it is the thing you are supposed to be proving! In this case, do not assume that the triangle has two equal angles: the definition of an isosceles triangle is that is has two equal *sides* (and you are supposed to **prove** that it has two equal angles).

(ii) Having proved that $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$, you don't have to start all over again to show that $\dfrac{\sin C}{c} = \dfrac{\sin B}{b}$; you can state that this second result follows by relabelling (i.e. swapping the letters around) or by symmetry.

(iii) Note that $BM = \cos\alpha$ and so $\sin A = \frac{BM}{1} = \cos\alpha$.

Preparation

2 **(i)** As a general rule, do not divide by something that might be zero. This is quite a simple example, but if you divide by x rather than factorising you will loose one of the solutions.

(ii) Remember to show key points on your sketch, but do not work out lots of values and plot the graph.

(iii) Note that these cubics (which have no x term) all have a turning point on the y-axis.

(iv) The key point here is that cubics have one turning point on either side of the x-axis if they have three real roots.

The STEP question

3 Find the coordinates of the turning points **before** you start considering the various cases.

For the second part, the letters are now a and b, not A and B, because in the first part there were conditions on the values of A and B.

You need to be careful with the direction of the implication here. The statement

"has three distinct real roots **if** $27b^2 + 4a^3b < 0$"
means
"**if** $27b^2 + 4a^3b < 0$ **then** the equation has three distinct real roots".

Some people showed that the converse statement is true, i.e.

"**if** the equation has three distinct real roots **then** $27b^2 + 4a^3b < 0$"

which is not what the question is asking for.

Start with the "if" statement. Factorise the inequality, so that you have:

$$\text{if} \quad 27b(b + \tfrac{4}{27}a^3) < 0$$

and from that you can consider the two cases that occur, (one case is $b > 0$; $(b + \frac{4}{27}a^3) < 0$). You can then draw a sketch for each and show that they do have three distinct real roots. You then need to repeat for the other inequality; four graphs are needed to cover all the possibilities (compare with $B > 0$, $A < 0$ in the first part of the question).

Some people started by considering the turning points, assuming that there are three roots. This is a dangerous approach, but can be made to work with careful use of "if and only if".

To do this, you might start by showing that:

the equation has three distinct real roots
if and only if
the turning points are on opposite sides of the y-axis

You will need to sketch graphs of cubics showing all the different cases to justify this. Once you have done this you can say:

the turning points are on opposite sides of the y-axis
if and only if
the y coordinates y_1, y_2 of the turning points have different signs
if and only if
$y_1 \times y_2 < 0$

and then use your y_1 and y_2 to complete the argument.

Warm down

4 Euclid did not have the SSS condition for congruence at this point in his book, so for this question we restricted ourselves to SAS.

If we do allow ourselves the SSS condition, then the easiest way to prove that an isosceles triangle has two angles is as in 1(i).

Start by showing that $\triangle BCD$ is congruent to $\triangle BAE$ by using SAS. You now know that $DC = AE$ and $\angle ADC = \angle AEC$ and can use SAS again with $\triangle ADC$ and $\triangle CEA$.

It may find it helpful to name the angles (for example: "Let $\angle BAE = \angle BCD = \beta$") and mark them on your diagram.