STEP CORRESPONDENCE PROJECT

Feedback: Assignment 3

Warm-up

1 **(i)** You can use the binomial expansion or multiply out the brackets by hand.

As long as you are careful (and remember for example that $(\sqrt{5})^3 = 5 \times \sqrt{5}$)) then you should be OK here.

Make sure that, whenever you have an equals sign, the things on either side are actually equal. For example, **don't** write

$$(3 + 2\sqrt{5})(3 + 2\sqrt{5})(3 + 2\sqrt{5}) = 9 + 6\sqrt{5} + 6\sqrt{5} + 20$$

(the right hand side is the result of expanding just two of the brackets); instead write

$$(3 + 2\sqrt{5})(3 + 2\sqrt{5})(3 + 2\sqrt{5}) = (9 + 6\sqrt{5} + 6\sqrt{5} + 20)(3 + 2\sqrt{5})\,.$$

Again, good practice is "one equals sign per line, equals signs aligned".

(ii) A few people just *stated* that the angle at the centre is twice the angle at the circumference, which is what we were asking you to *prove*!

It is important not to assume anything about the triangle APB that you are not told; some people, for example, assumed that the triangle is equilateral. It is a good idea to make sure your diagram shows a triangle that is not special in any way.

Can you prove the case where $\angle APB$ is obtuse? Do you actually need to produce a new proof for this case?

Preparation

2 **(i)** Two points to watch out for here.

(a) You have to be careful about wording. In this part of the question, x could lie in one of two ranges, so you should write $x < a$ **or** $x > b$ (say) rather than $x < a$ **and** $x > b$, as any single value of x can only be in one range. Of course, we know what you mean, but it is good to be precise.

(b) You have to be careful to use the appropriate type of inequality — if the question involves **strict** inequalities ($<$, $>$) then the answer should involve strict inequalities, and similarly for **weak** inequalities ($\leq$, $\geq$).

(ii) You don't know that the roots are going to be integers; but if they are integers there are not many possibilities (they can only be factors of 6), so it might have been worth testing these. Most people, however, divided the cubic by the known factor $(x - 3)$ and then factorised the resulting quadratic, which was rather impressive!

Don't forget to answer the question — it asks you to find the roots so you do actually need to write "$x = \cdots$".

(iii) The wording here put some people off. First factorise the quadratic into two brackets. One bracket or the other (or both) must be zero, which gives the equations of two straight lines. Sketch these and then you are done — nothing more complicated needed.

(iv) You should already have a diagram showing the two relevant lines from the previous part of the question.

There are two main approaches here.

(a) You can write the expression as the product of two linear factors and then consider the signs of each bracket, which will result in two inequalities for each case (for example, $x - 2y \geqslant 0$ and $x - y \leqslant 0$ for the first bracket to be positive and the second bracket negative).

(b) A perhaps simpler line of attack is to draw the two straight lines (as in part (iii)) and note that they divide the plane into 4 regions. You then only need to determine the sign of the quadratic expression in each of the 4 regions, for example by calculating its value at a single point in each region.

Note that there are only 4 regions — the x and y axes are not boundaries of regions. It is not a problem if you test in 8 regions rather than 4; you are just making life harder for yourself!

The STEP question

3 **(i)** Generally people found that the preparation they had already done meant that they could knew how to approach this question.

Again you have to be careful to distinguish between **strict** or **weak** inequalities.

(ii) A few people went through the process of factorising the cubic expression (in x and y) (and did it well — there was some complicated algebra in places) but most just used part (i) to help them, essentially just replacing 1 with y.

(iii) Some people considered where the product of the three factors was positive (i.e. where all three are positive, or where two are negative and one positive) whilst others tested points in the regions to see where the inequality was true. Both ways are fine (if well argued) **but** you do need to show some working to justify your solutions.

Warm down

4 **(i)** You need to think very carefully about the question. It says that **if** there is an even number **then** the other side is a vowel. It does not follow that if there is a vowel on one side then there must be an even number on the other side; furthermore if there is an odd number on one side there can be any letter on the other side.

It is fairly obvious that we must check the card showing the 6. There is one other card that we must check, and this card could show the statement to be false. Think about the different implications there might be depending on what is on the second sides of the cards.

(ii) If you are stuck you can start by trying to work out how to time 30 mins. Or you can google it, but please not before you have had a good think about it.