STEP CORRESPONDENCE PROJECT

Postmortem: Assignment 4

Warm-up

1 **(i)** Remember that although $\cos^2\theta + \sin^2\theta = 1$, it is **not** generally the case that $\cos\theta + \sin\theta = 1$ (for what values of $\cos\theta$ does this equality hold??).

(ii) As we have said before, a good, clear diagram is invaluable.

Make sure that you don't assume too much: in this question, it is OK to assume that $x > a$ (or vice versa); but in other situations you might have to consider the two cases separately. You certainly can't assume that $x = 0$ (so that A is on the y-axis) because that would be restricting yourself to a right-angled triangle.

(iii) This was just an exercise in manipulating surds and was answered well by everyone.

Preparation

2 **(i)** Note that the bit in italics said *leave your answers as fractions*. This implies that you should **not** be working with decimals. There is actually no need to find angle C, you can get $\sin\theta$ from the value of $\cos\theta$ and the result you demonstrated in question 1(i).

Note also that we ask for the *value*, not the *exact value*; we don't have to use the word *exact* (even though we want the exact value) because asking for the *value* is unambiguous. There is no need to redefine a perfectly good word in the English language just because lots of people use calculators and only ever find *approximate* values. End of rant.

(ii) This part provides a simple example of the method required for the main part of the STEP question; it was well done.

(iii) By definition $\sqrt{x}$ is positive or zero (i.e. it is non-negative). So $\sqrt{9} = 3$ and not -3, and $\sqrt{(-5)^2} = 5$. This means that $\sqrt{(x^2)} = x$ if x is positive but $\sqrt{(x^2)} = -x$ if x is negative.

The STEP question

3 **(i)** Some people forgot that the base is a triangle; otherwise this bit was found to be straightforward.

(ii) Draw a good, clear diagram.

You must be careful here not to use a for two different things. In the question, a is defined to be the length OA. In the Cosine Rule you usually use a for the length BC but as a has already been used you have to choose a different letter for BC (or just call it BC). Writing the Cosine Rule as $AB^2 = BC^2 + AC^2 - 2 \times BC \times AC \times \cos\theta$ will help to avoid confusion.

Once you have found $\cos\theta$ you can find $\sin\theta$ using the result from question 1(i), and then find the area of triangle ABC using $\frac{1}{2} \times BC \times AC \sin C$.

Equating the volume of the tetrahedron as found in the first part of the question to $\frac{1}{3} \times (\text{area of } ABC) \times d$ followed by some squaring and a bit of manipulation should get you to the required result.

Warm down

4 **(i)** You need to show that 2 socks are not enough to ensure a pair (by providing an example), *and* that 3 socks are enough to ensure a pair.

(ii) Similar comment as above: there are again two parts to the question.

(iii) Lots of people just wrote down the answer $(2n + 1)$ without any justification; the justification rather than the answer is really the important part of the question (no one takes socks from a drawer at random).

$2n$ socks would be enough if there are an even number of blue socks and an even number of red socks.

But if there are an odd number of red socks and an odd number of blue socks $2n$ socks are not enough. You can show this by letting the number of red socks you pick be $2r+1$ and the number of blue socks be $2b+1$, in which case you have only $n-1$ pairs. But you do have a spare red and a spare blue sock, so you can then consider what happens when you add on one more sock.

A justification on the above lines would be acceptable (with the gaps filled in) but there are many other ways of thinking about it.